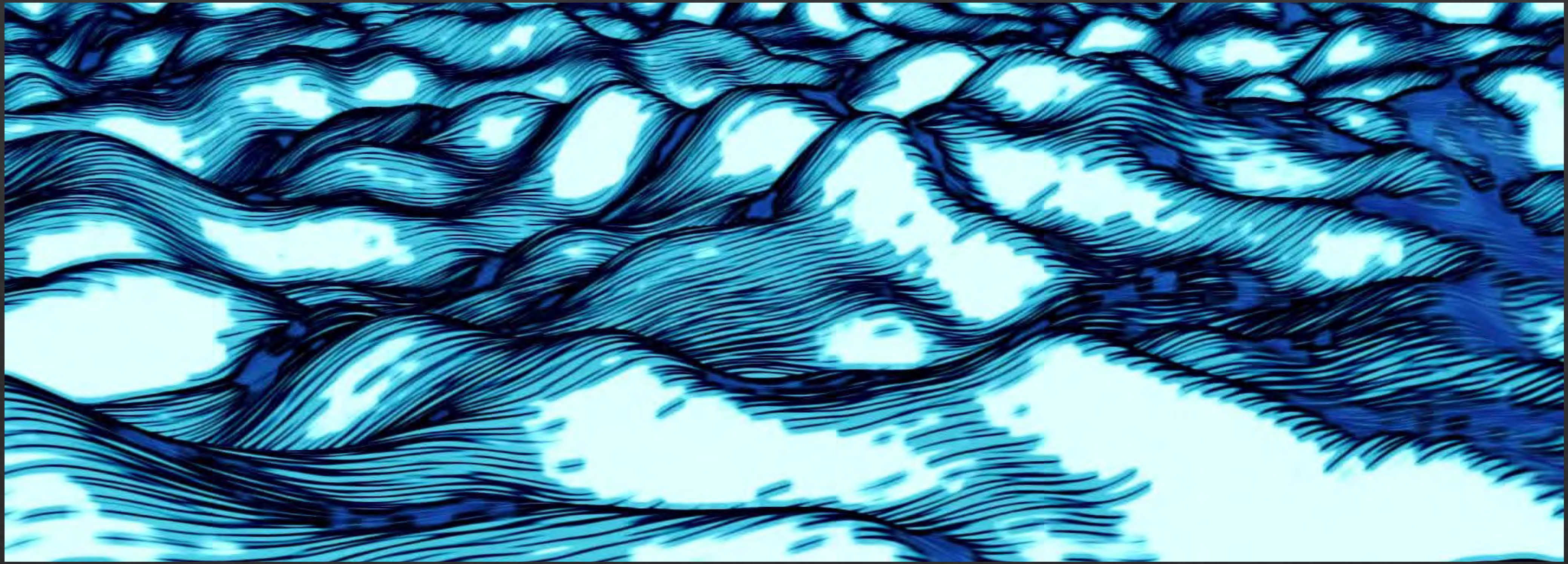
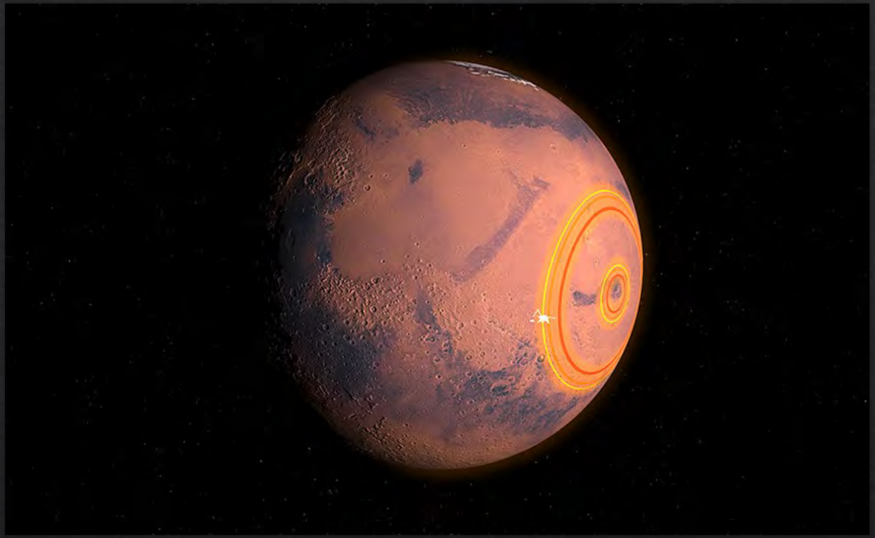


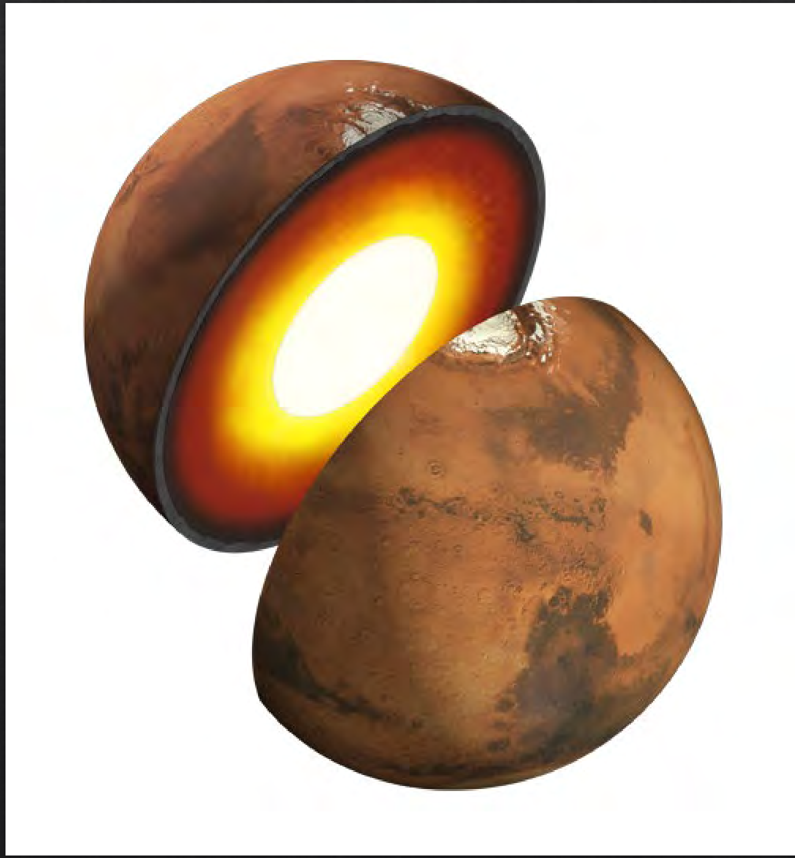
Simulating Seismic Waves in Spherically Symmetric Mediums

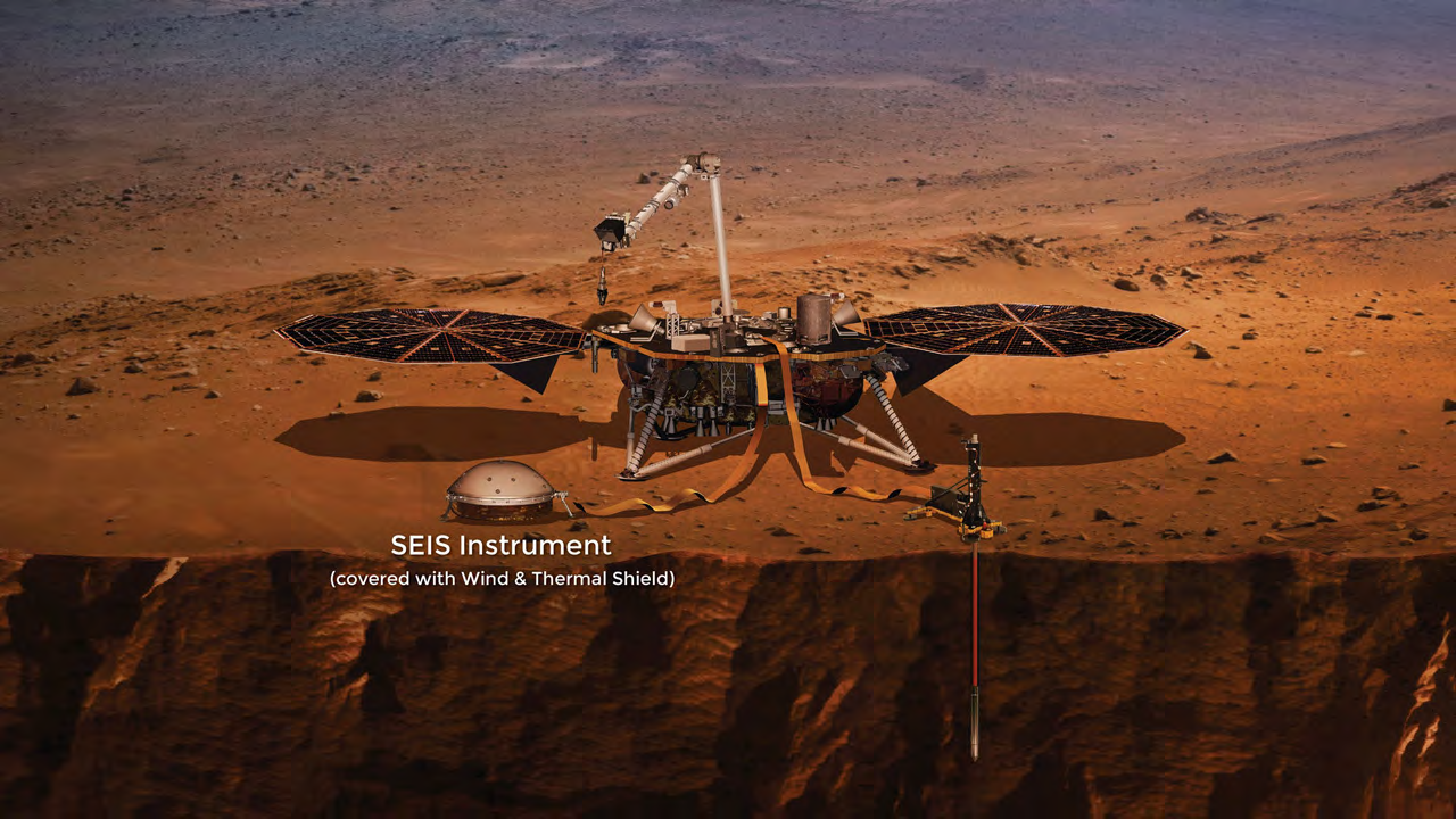
Presented by Andrew Wong

Professor: Vitaly Katsnelson

Interior Imaging using Seismic Waves






SEIS Instrument
(covered with Wind & Thermal Shield)

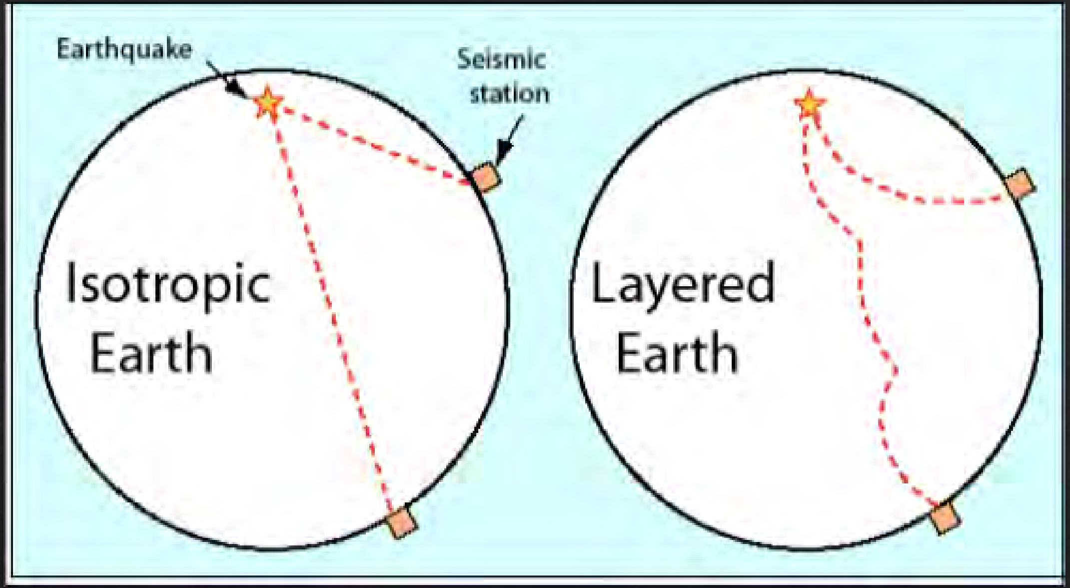
Background, Problem, and Question

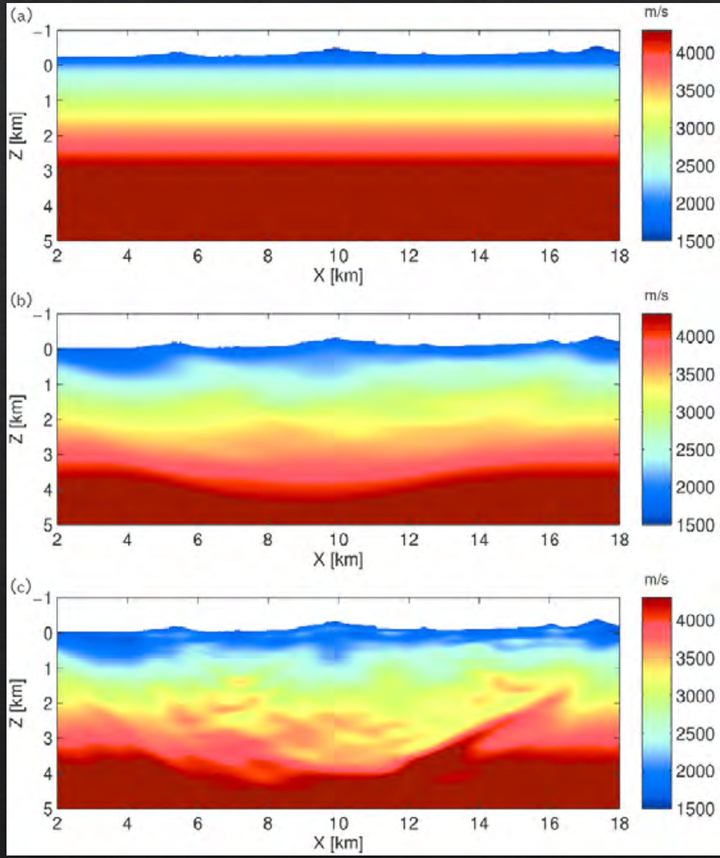
The interior of a planet can be imaged by using seismometers to analyze how waves travel underneath its surface

However, it is expensive to transport and install multiple seismometers

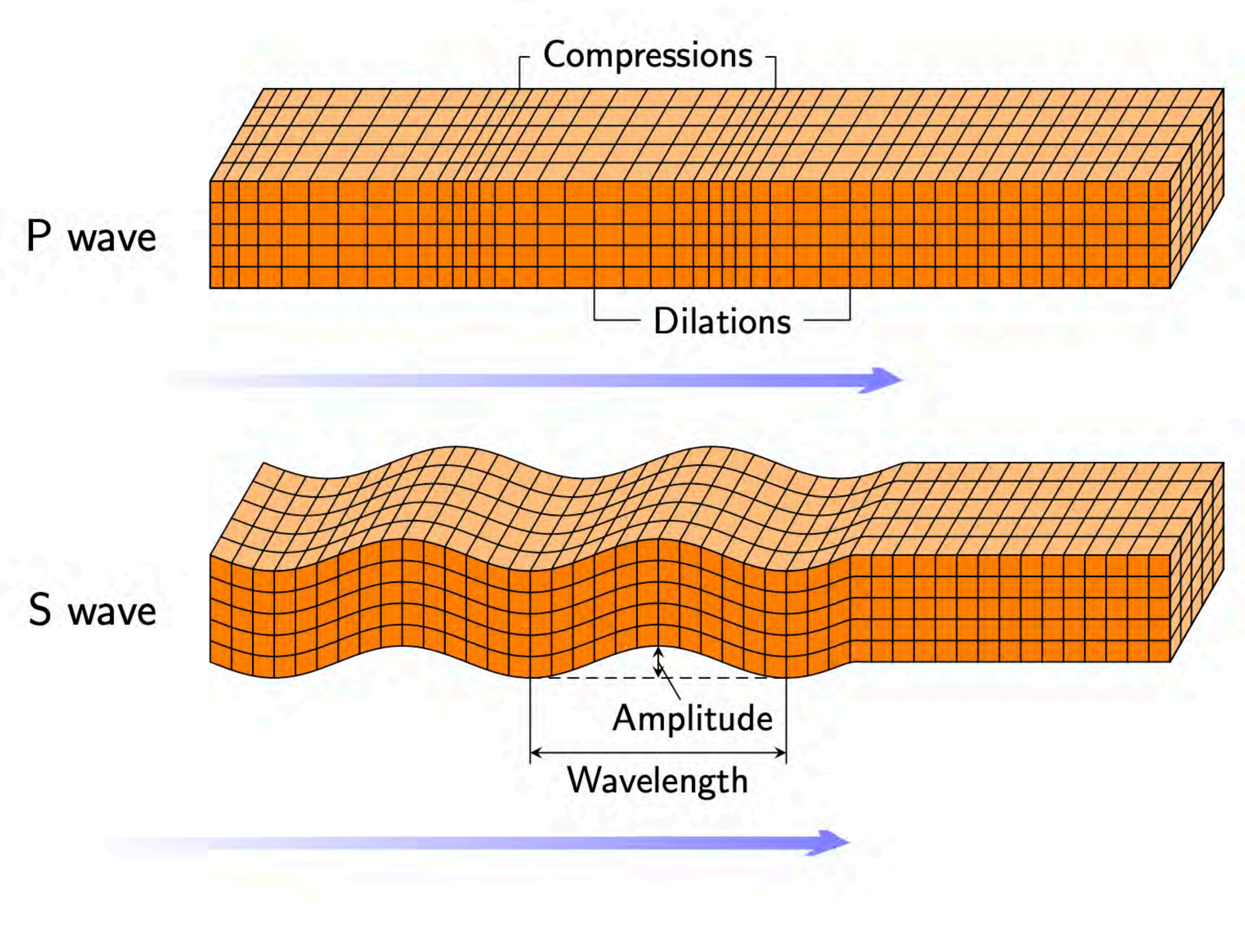
Can we model the interior of another planet using just one seismometer?

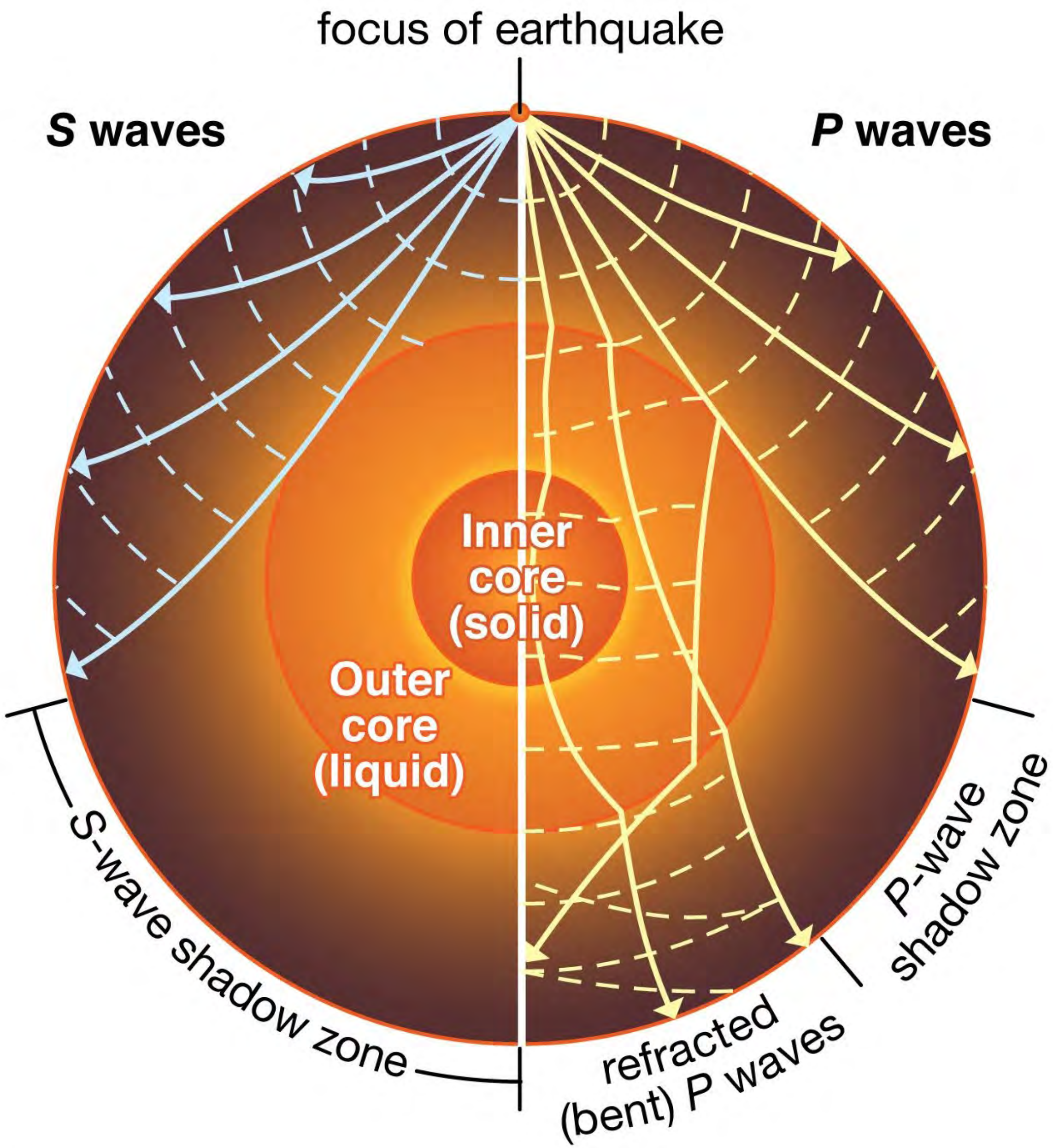
Travel-time Tomography





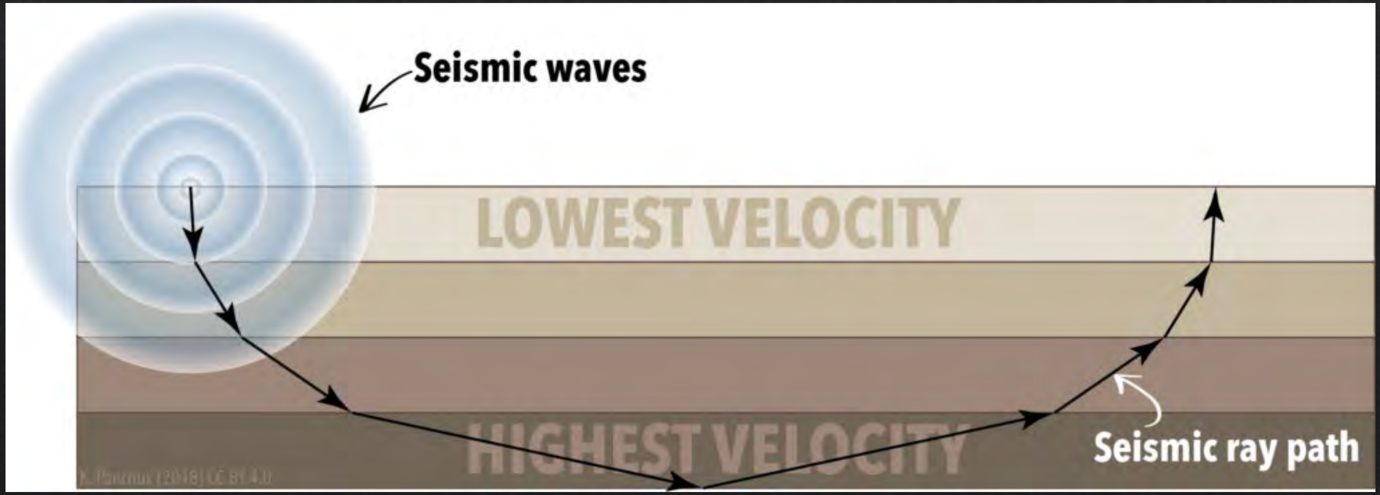
Types of Waves




focus of earthquake
S waves
P waves
Inner core (solid)
Outer core (liquid)
S-wave shadow zone
P-wave shadow zone
refracted (bent) P waves

The Physics of Seismic Waves

- Seismic waves follow the fastest path
- Ray paths show the travel path of a specific part of the wave
- At layer interfaces, waves either transmit (refract) or reflect
- Mode conversion – type of wave may change upon reaching an interface
- Periodic waves eventually return to the surface at the same point they originate from



Hamilton's Equations of Motion

- Accurately modeling rays is complex
- These equations can be simplified when we assume spherical symmetry
- The symmetry means we only need to consider the radius

$$\frac{dr}{ds} = v p_r$$

$$\frac{d\theta}{ds} = \frac{v}{r} p_\theta$$

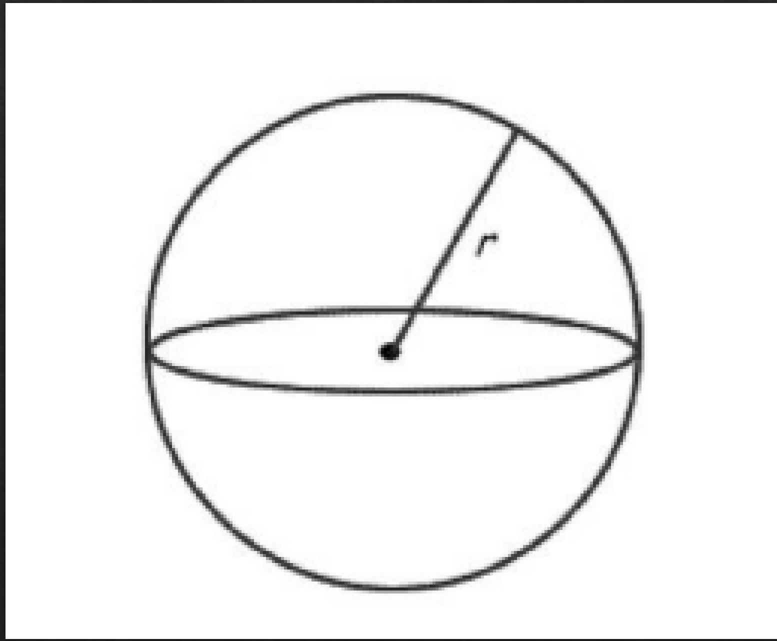
$$\frac{d\varphi}{ds} = \frac{v}{r \sin\theta} p_\varphi$$

$$\frac{dp_r}{ds} = \frac{v}{r} ({p_\theta}^2 + {p_\varphi}^2) - \frac{1}{v^2} \frac{\partial v}{\partial r}$$

$$\frac{dp_\theta}{ds} = -\frac{v}{r} (p_r p_\theta - \cot\theta\, {p_\varphi}^2) - \frac{1}{v^2 r} \frac{\partial v}{\partial \theta}$$

$$\frac{dp_\varphi}{ds} = -\frac{v}{r} (p_r p_\varphi + \cot\theta\, p_\theta p_\varphi) - \frac{1}{v^2 r \sin\theta} \frac{\partial v}{\partial \varphi}.$$

Model Simplification using Spherical Symmetry



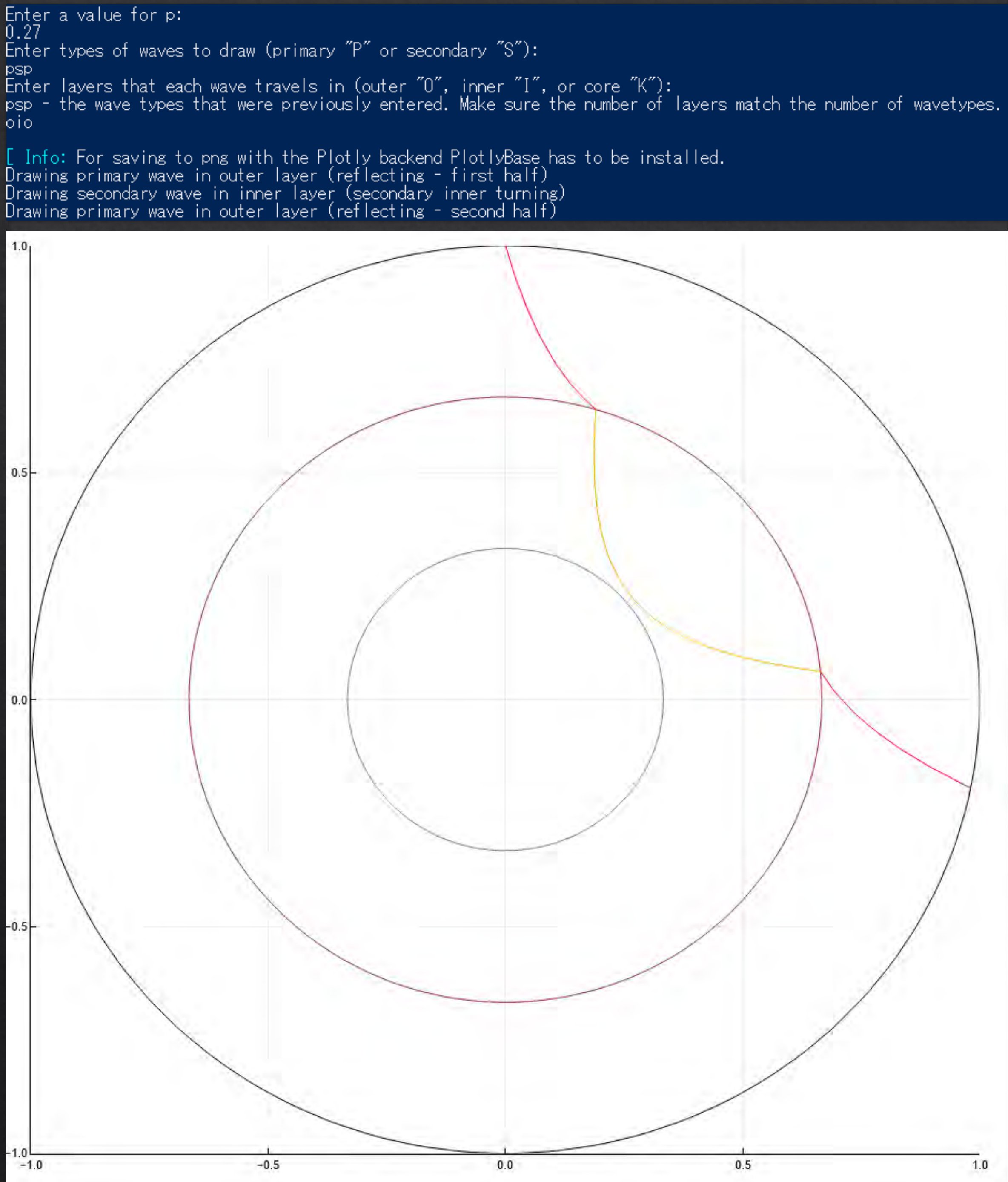
$$c(x, y, z) \to c(r)$$

$$\frac{d}{dr}\left(\frac{r}{c(r)}\right) > 0, r \in [0, 1].$$

$$c_p(r), c_s(r)$$

Program Overview

- Three layers – outer, inner, core
- Two wave types – primary (red) and secondary (yellow)
- Program draws waves according to user's input
- Pictured: PSP wave passing through layers OIO with a p-value of 0.27

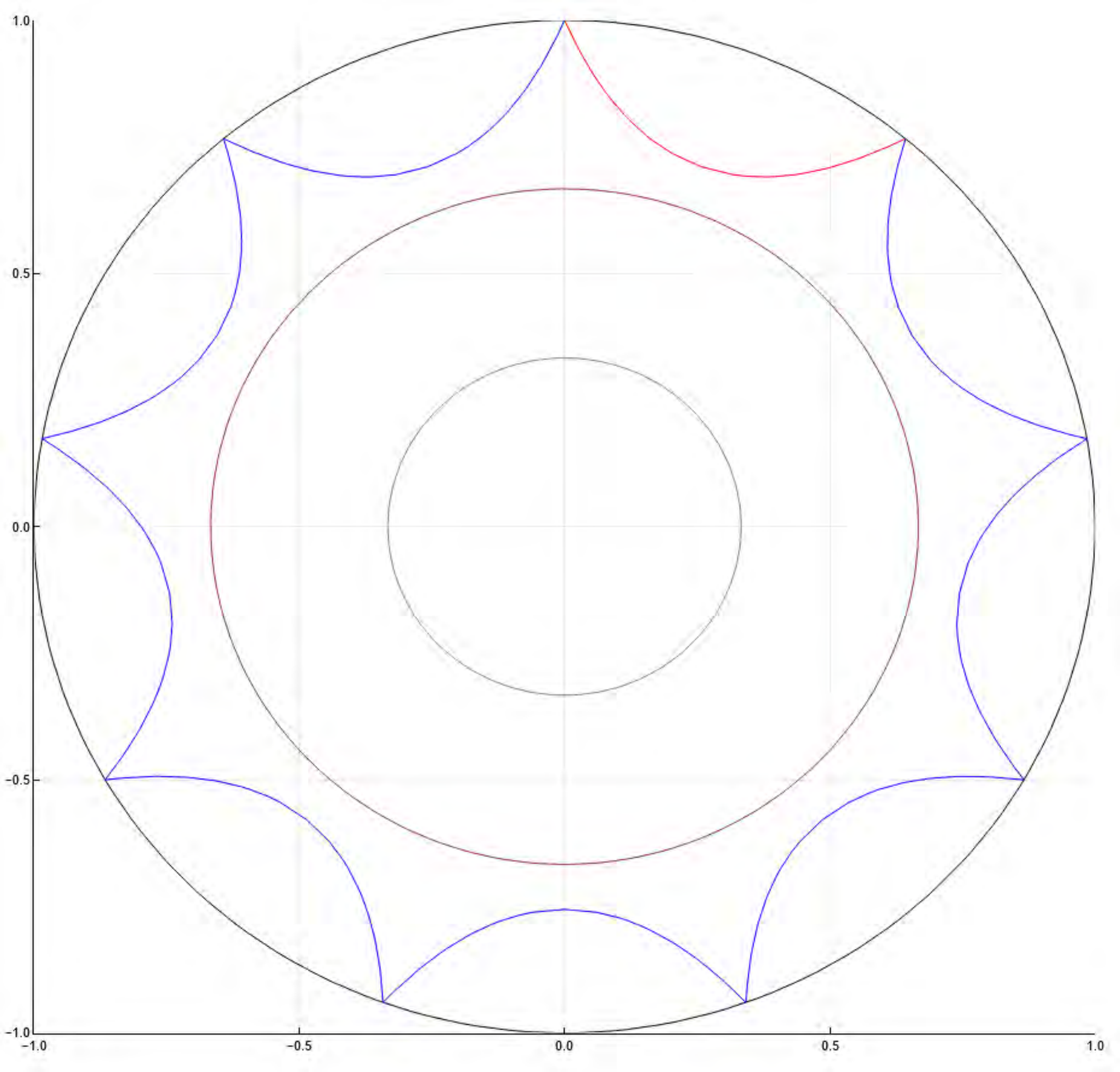


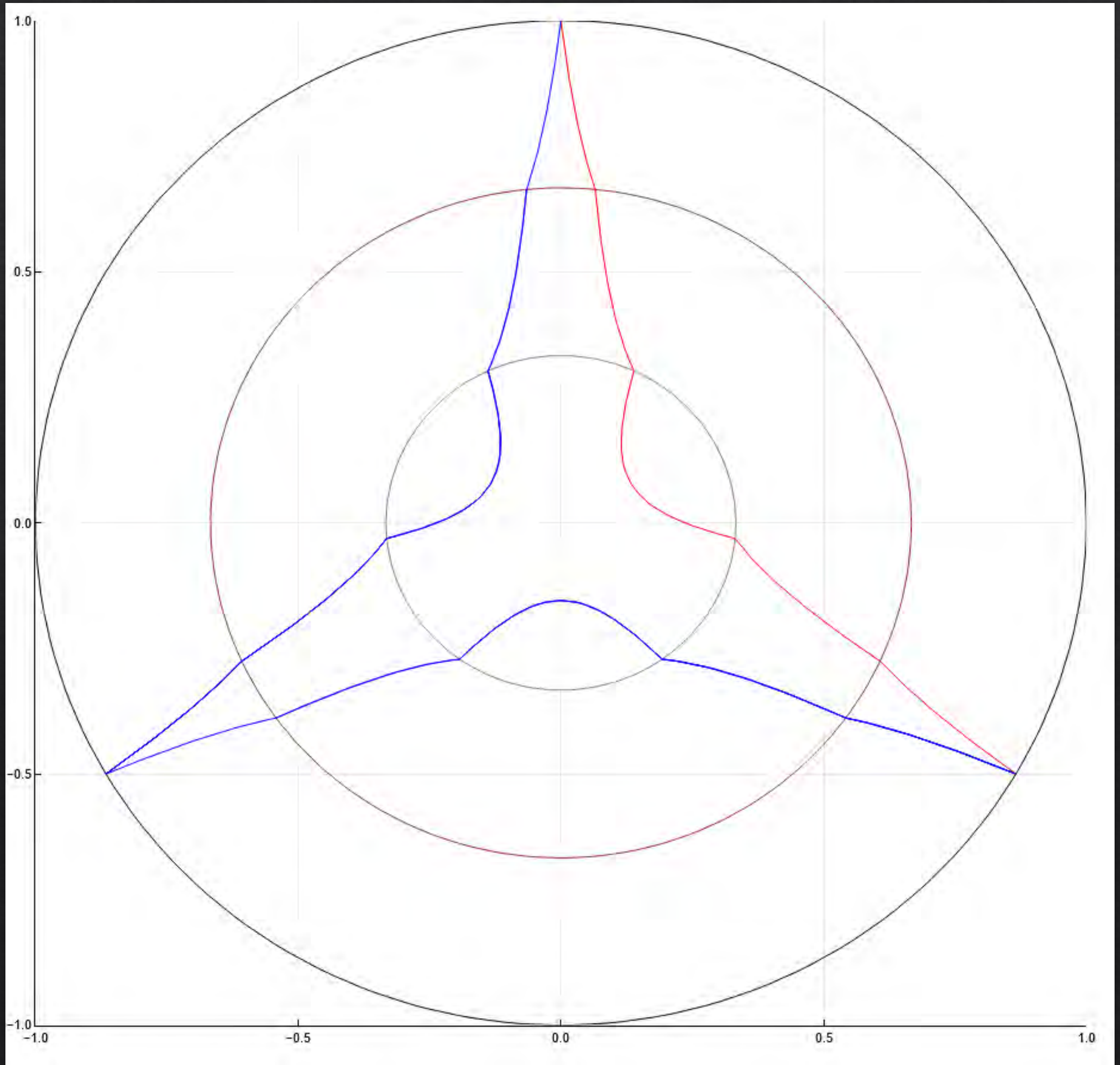
Calculating Closing Waves

```
if (findClosingPaths)
    windingNum = 1    # n
    numPatterns = 10   # m, total number of times to draw pattern

    drawPath = "o"
    closingPath(p) = numPatterns / (2 * pi) * getEpicentralDistance(p, drawPath) - windingNum

    println("Enter initial guess: ")
    initialGuess = readline()
    initialGuess = parse(Float64, initialGuess)
    println(find_zero(closingPath, initialGuess))
end
```

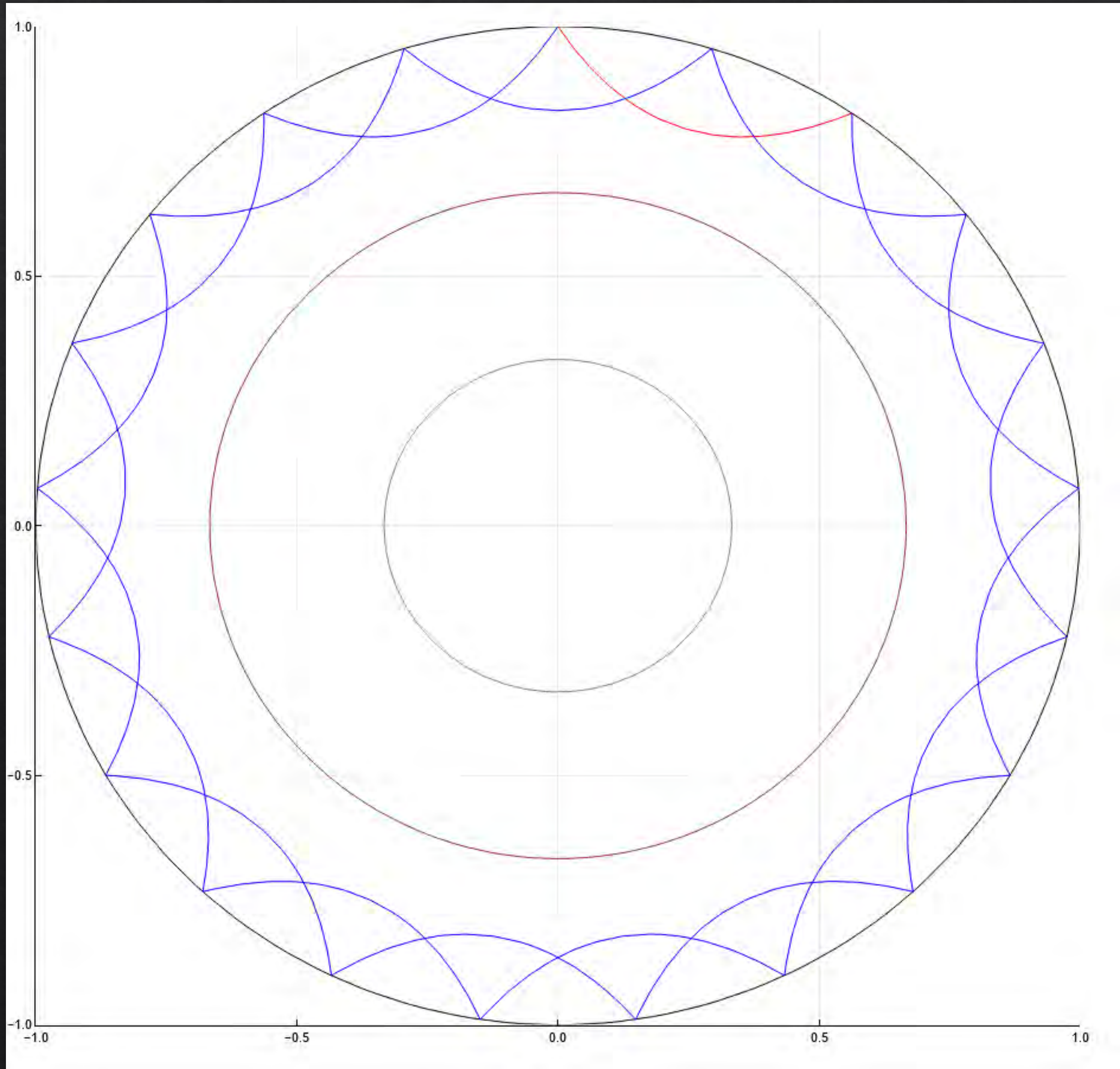
```
distance = 0
if (numOuterPaths > 0)
    if (p > outerGrazingP)
        println("smooth turning outer paths")
        outerAlpha = getAlpha(p, outerInterface, surface, 'o', 'p')
        distance += 2 * numOuterPaths * outerAlpha
    else
        println("reflecting outer paths")
        outerAlpha = getAlpha(p, outerInterface, surface, 'o', 'p')
        distance += numOuterPaths * outerAlpha
    end
end
if (numInnerPaths > 0)
    if (p > innerGrazingP)
        println("smooth turning inner paths")
        innerAlpha = getAlpha(p, innerInterface, outerInterface, 'i', 'p')
        distance += 2 * numInnerPaths * innerAlpha
    else
        println("reflecting inner paths")
        innerAlpha = getAlpha(p, innerInterface, outerInterface, 'i', 'p')
        distance += numInnerPaths * innerAlpha
    end
end
if (numCorePaths > 0)
    coreAlpha = getAlpha(p, 0, innerInterface, 'k', 'p')
    distance += 2 * numCorePaths * coreAlpha
end
return distance
```

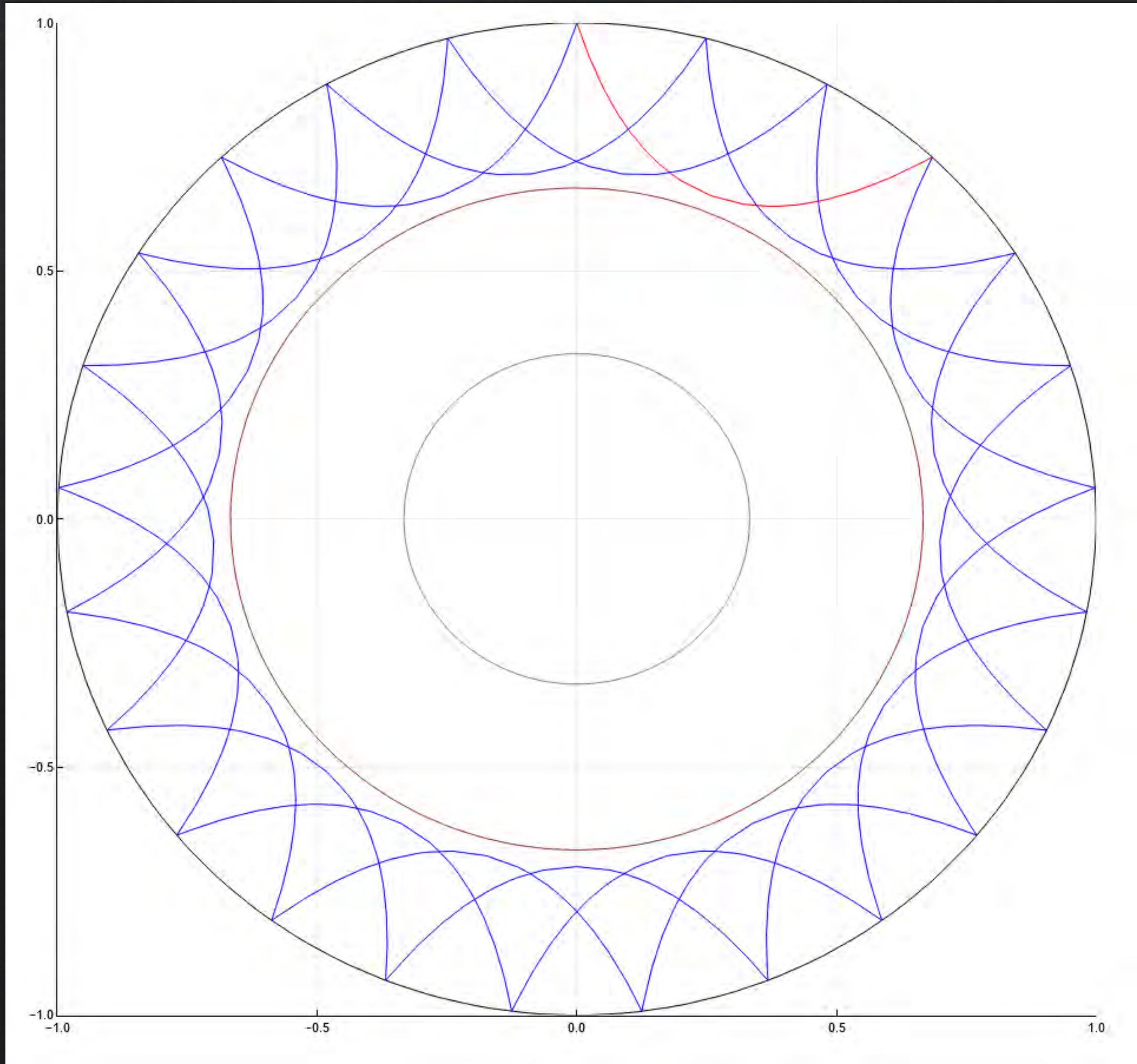
Sample Output for Closing Waves





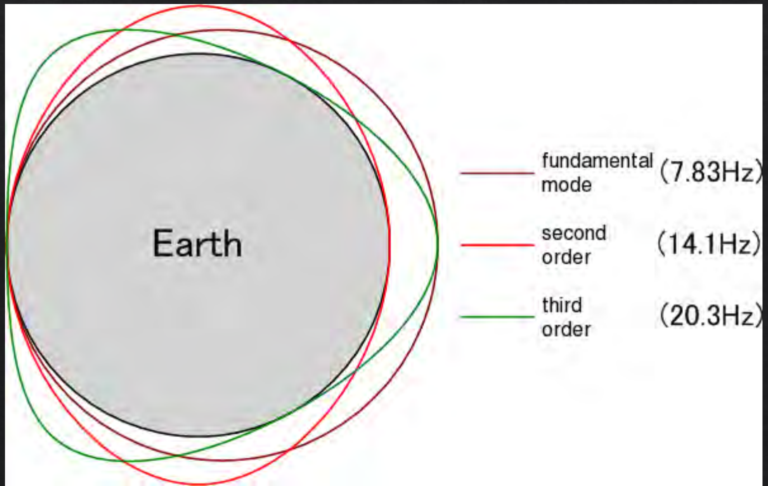
More Closing Waves

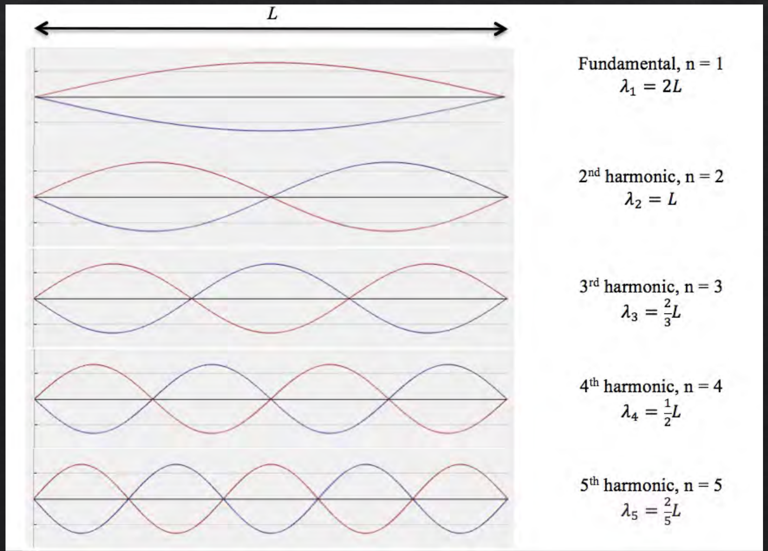




Concluding Thoughts & Future Work

- Many periodic waves exist with measurable travel times
- Tomographic images can be produced with just a single seismometer and enough of these periodic waves
- Further improvements: calculating the periods of these waves
- Connecting the natural frequencies of oscillations of the planet to these periods
- Experimentally verifying the connection between these periods and the planet's natural frequencies of oscillation





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- https://mars.nasa.gov/resources/22734/seismic-waves-inside-mars/
- https://www.britannica.com/science/secondary-wave
- https://openpress.usask.ca/physicalgeology/chapter/3-2-understanding-earth-through-seismology-2/
- https://mars.nasa.gov/insight/spacecraft/instruments/seis/
- https://www.researchgate.net/figure/aInitial-velocity-model-for-traveltime-tomography-binverted-velocity-model-by-traveltime_fig3_306292093
- https://en.wikipedia.org/wiki/Schumann_resonances
- https://phys.libretexts.org/Bookshelves/Waves_and_Acoustics/